MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2022/2023

PEM0046 – INTRODUCTION TO PROBABILITY AND STATISTICS

(Foundation in Engineering)

5 OCTOBER 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of **FIVE** printed pages including the cover page.
2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided. All necessary working steps **MUST** be shown.
4. Statistical table is provided.

Answer ALL questions [100 marks].

QUESTION 1 [25 marks]

a. Two six-sided dice, one white and one black, are rolled together. Let X represent the difference between the highest and lowest scores on the dice.

i. What are the possible values of X? [2 marks]

ii. Tabulate the probability distribution of X. [5 marks]

iii. Calculate the mean value of X. [3 marks]

iv. What is the probability of obtaining a difference of at least 3 when rolling the two six-sided dice? [3 marks]

v. If the white die is replaced with an eight-sided blue die, numbered from 1 to 8, while keeping the black die, what are the new possible values of X? [2 marks]

b. A continuous random variable X has probability density function.

$$\mathrm{f}(x) = \begin{cases} mx - \frac{3}{4}x^2 & , \quad 0 \leq x \leq 2 \\ 0 & , \quad \text{otherwise} \end{cases} \quad ; \quad m \text{ is a constant.}$$

i. Show that $m = \frac{3}{2}$. [3 marks]

ii. Find the cumulative distribution function of x, $\mathrm{F}(x)$.
Hence, find $\mathrm{P}\left(\frac{2}{3}m < X < 3\right)$. [7 marks]

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QUESTION 2 [25 marks]

a. The microchips manufacturing process involves multiple stages, lasting months from design to production. Consider a factory manufacturing microchips of which 25% are expected to be defective. A sample of 12 microchips is randomly selected, find the probability of obtaining:

i. at most 3 defective microchips. [1 mark]

ii. from 2 to 4 defective microchips. [2 marks]

iii. between 6 and 9 **non-defective** microchips. [2.5 marks]

iv. Find the mean and standard deviation for **non-defective** microchips. [2 marks]

v. A random sample of n number microchips is drawn. To ensure a probability of 0.9 or higher in finding at least one defective chip from the n number of microchips, what is the smallest n required? [3.5 marks]

b. Aisyah is a secretary at ABC company. She receives an average of 18 phone calls per hour. Using a Poisson distribution, find the probability that

i. in a 30-minute period, there will be more than 20 phone calls received. [1.5 marks]

ii. exactly 10 phone calls received in 1-hour and at least 15 phone calls received in the next hour. [3.5 marks]

c. The time needed to complete the quiz of a certain subject is normally distributed with a mean of 32 minutes and a standard deviation of 6 minutes. If a student is selected at random, find the

i. probability that the student chosen will finish the quiz in less than 30 minutes. [2.5 marks]

ii. probability that the student chosen will finish the quiz in more than 25 minutes and less than 40 minutes. [3.5 marks]

iii. How much time should the lecturer allocate for the quiz to ensure that 90% of the students complete it on time? [3 marks]

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QUESTION 3 [25 marks]

a. Cindy conducted a taste test for various cake flavours. While there are numerous options available, she intends to buy only three different flavours. Below are the prices (in RM) of all five available cake flavours at the Delicious Bakery Shop.

Cakes Menu	Price (RM)
Moist Chocolate Cake	98
Chocolate Fudge	90
Black Forest	100
Absolute Durian	130
Chocolate Strawberry	120

i. Construct a sampling distribution of sample mean price with samples of size three of different flavours of cakes which can be selected without replacement from this population. Leave your answer in 2 decimal places. [8 marks]

ii. Find the mean and the standard deviation of the sample mean price.

[Hint: $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$] [5 marks]

iii. Cindy takes a random sample of four different flavours of cakes from this population which includes the prices of RM 98, RM 90, RM 100, and RM 130. Calculate the non-sampling error if Cindy mistakenly recorded the last price as RM 150 instead of RM 130. [2 marks]

b. An automotive battery, or car battery, jump-starts a motor vehicle by providing electric current to the starting motor. From the past record, the lifetime of a certain type of car battery manufactured by ABC company is normally distributed with a mean (μ) of 500 days and a standard deviation (σ) of 61 days. A random sample of 40 car batteries is drawn from this population.

i. What is the probability that the mean lifetime of these car batteries is more than 510 days? [3 marks]

ii. What is the probability that the mean lifetime of these car batteries within 30 days from the population mean (μ)? [3 marks]

iii. To ensure replacing no more than 2 % of all manufacturer car batteries, what warranty duration should ABC company provide? [4 marks]

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QUESTION 4 [25 marks]

A fitness instructor is keen on understanding the potential connection between the resting heart rates of their clients and the number of workout sessions they attend. To investigate this relationship, they collected data over the course of several weeks. The following data represents the number of workout sessions attended by various clients and their corresponding resting heart rates (in beats per minute):

Client	Ada	Ben	Cam	Dot	Eva	Fed	Geo	Ham
Workout sessions (X)	10	8	6	15	12	5	9	11
Resting Heart Rate (Y)	68	72	75	62	65	80	70	67

a. Draw a scatterplot with the number of workout sessions on the x-axis and resting heart rate on the y-axis on the graph paper provided. Describe the pattern you observe. [3 marks]

b. Calculate the correlation coefficient between the number of workout sessions and resting heart rates. Interpret the values of the correlation coefficient in terms of the strength and direction of the relationship. [12 marks]

c. Find the equation of the least squares regression line for the provided data. Hence, draw the least squares regression line on the same graph paper as the scatterplot from part (a). [6 marks]

d. For a client with a resting heart rate of 69 beats per minute, estimate the number of workout sessions they might have attended (round the answer to the nearest integer). [2 marks]

e. Interpret the slope of the least square regression line (from part c) in the context of the problem. Explain what the slope value indicates about the relationship between resting heart rates and workout sessions. [2 marks]

[Round your answers to 3 decimal places for Question 4]

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